Migration in the equivalent offset domain

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Bancroft et al. (1994) have introduced equivalent offset migration(EOM) as a prestack time migration method that, according to their description, is related to Kirchhoff prestack time migration. Margrave et al. (1998) have given a derivation of a similar method (equivalent wavenumber migration) which is implemented in the Fourier domain. The published details of EOM, which is implemented in the time-space domain, have been scarce. A derivation of EOM from Kirchhoff migration is presented here, which can be used as the basis of a correct implementation. During the presentation these results will be examined to see the computational benefits and limitations of this form of migration.

The derivation of EOM from prestack Kirchhoff migration consists of a change of variables, followed by a rearrangement of terms. In a common-shot configuration, the equation for the 2D acoustic prestack Kirchhoff time migrated wavefield, $P(x,\tau)$, in terms of the recorded wavefield, $p(h,y,t)$, is (Docherty, 1991):

$$P(x,\tau) = \rho(\tau) * \iiint\limits_{t\,y\,h} \frac{\cos\phi}{v_G} \frac{A_G(x,\tau,y+h)}{A_S(x,\tau,y-h)} p(y,h,t)\delta(t-\tau_1)dh\,dy\,dt,$$

where, as indicated in Figure 1, (x,τ) are the migrated position and time, and (y,h,t) are the distance from input trace midpoint to x, the half-offset, and the recorded time, respectively. $\rho(\tau)$ is the half-differential convolution operator, ϕ is the angle between the normal of the surface and the ray emerging at the receiver, and v_G is the velocity at the receiver. $A_{G,S}$ are the WKBJ ray amplitudes from the receiver and shot to the imaging point. Constant factors of 2 or π have been ignored in this equation. In time migration, v_G is usually taken to be constant, and it is assumed that $A_G = A_S$. The above equation is an integration over a two-dimensional surface known as Cheop's pyramid:

$$\tau_1(y,h) = (\frac{\tau^2}{4} + \frac{(y+h)^2}{v^2})^{1/2} + (\frac{\tau^2}{4} + \frac{(y-h)^2}{v^2})^{1/2},$$

where v is velocity. At constant time, we now transform from the (h,y) domain to the (h_e,ξ) domain, where the new coordinates are chosen for the following reasons. Bancroft et al. (1994) have defined the equivalent offset, h_e, as

$$h_e^2 = h^2 + y^2 - 4h^2y^2/v^2\tau_1^2.$$

Coordinate h_e is chosen so that the traveltime equation becomes the equation of a hyperbola, independent of ξ:

$$\tau_1(y,h) = \tau_2(h_e) = (\tau^2 + \frac{4h_e^2}{v^2})^{1/2}.$$

The requirement that h remain real implies that either $y < h_e$ and $y^2 > v^2\tau^2/4$ or $y > h_e$ and $y^2 < v^2\tau^2/4$. Coordinate ξ is arbitrary, as long as it is oblique to the h_e axis in the (h,y) plane (obliquity ensures that the Jacobian exists). Obvious choices for ξ are either y or h. Alternatively, the orthogonal curvilinear coordinates (h_e,θ) could be used, where θ is the coordinate that is always orthogonal to h_e in the (h,y) plane. θ is given by

$$\tan\theta = \frac{h}{y}\exp\{\frac{2}{v^2\tau_1^2}(y^2-h^2)\}.$$

Notice that when the total traveltime, τ_1, becomes large, a slice at constant time through Cheop's pyramid becomes circular, and transforming from (h, y) to (h_e, θ) reduces to a transformation from Cartesian to polar coordinates. Making the change of variables to (t, h_e, ξ) in the simplified Kirchhoff integral equation, and recognizing that τ_2 is independent of ξ (so the delta function comes outside the ξ integral), we get

$$P(x,\tau) = \rho(\tau) * \int_t \int_{h_e} \delta(t-\tau_2) \int_\xi \cos\phi \; p'(h_e,\xi,t) \left| \frac{\partial(h,y)}{\partial(h_e,\xi)} \right| d\xi \, dh_e \, dt.$$

Since ϕ will usually be, to a good approximation, independent of ξ, we can now write

$$P(x,\tau) = \rho(\tau) * \int_t \int_{h_e} \cos\phi \; q(h_e,t) \delta(t-\tau_2) dh_e dt \,,$$

where

$$q(h_e,t) = \int_\xi p'(h_e,\xi,t) \left| \frac{\partial(h,y)}{\partial(h_e,\xi)} \right| d\xi \,.$$

This last pair of equations expresses the procedure for migrating by equivalent offset. We recognize $q(h_e,t)$ as the common-scatter-point (CSP) gather for a particular migrated output location, *x.* According to the last equation, calculation of a CSP gather consists of a weighted stack of the transformed wavefield at constant h_e and t. Once the CSP gather is calculated, all that remains is a normal Kirchhoff migration integral.

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Docherty, P., 1991, A brief comparison of some Kirchhoff integral formulas for migration and inversion: Geophysics, **56**, 1164-1169.

Margrave, G.F., Bancroft, J.C., and Geiger, H.C., 1998, The theoretical basis for prestack migration by equivalent offset: submitted to Geophysics.

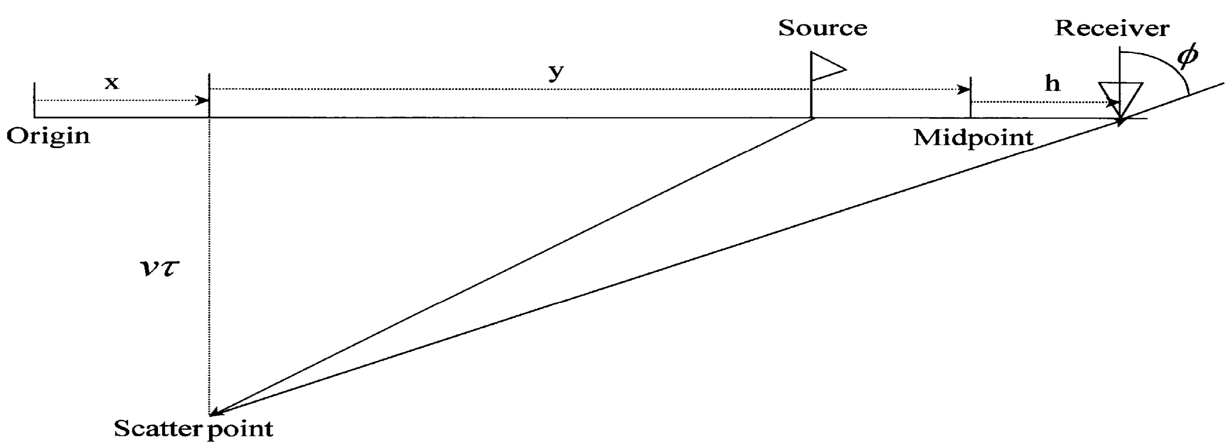

Figure 1: Geometry of equivalent-offset migration